



FORMULARIOS BÁSICOS Y TABLAS DE INTEGRACIÓN

ÁLGEBRA

Ceros y factores de un polinomio

Sea $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ un polinomio. Si $p(a) = 0$, entonces a es un *cero* del polinomio y una solución de la ecuación $p(x) = 0$. Además, $(x - a)$ es un *factor* del polinomio.

Teorema fundamental del álgebra

Un polinomio de grado n tiene n ceros (no necesariamente distintos). Aunque todos estos ceros pueden ser imaginarios, un polinomio real de grado impar tendrá por lo menos un cero real.

Fórmula cuadrática

Si $p(x) = ax^2 + bx + c$ y $0 \leq b^2 - 4ac$, entonces los ceros reales de p son $x = (-b \pm \sqrt{b^2 - 4ac})/2a$.

Factores especiales

$$x^2 - a^2 = (x - a)(x + a)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$$

Teorema del binomio

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \dots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \dots \pm nxy^{n-1} \mp y^n$$

Teorema del cero racional

Si $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ tiene coeficientes enteros, entonces todo *cero racional* de p es de la forma $x = r/s$, donde r es un factor de a_0 y s es un factor de a_n .

Factorización por agrupamiento

$$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$$

Operaciones aritméticas

$$ab + ac = a(b + c)$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \frac{ad}{bc}$$

$$\left(\frac{a}{b}\right)\frac{c}{c} = \frac{a}{b}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c}$$

$$\frac{ab + ac}{a} = b + c$$

Exponentes y radicales

$$a^0 = 1, \quad a \neq 0$$

$$(ab)^x = a^x b^x$$

$$a^x a^y = a^{x+y}$$

$$\sqrt[n]{a} = a^{1/n}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$\sqrt[n]{a} = a^{1/n}$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$\sqrt[n]{a^m} = a^{m/n}$$

$$a^{-x} = \frac{1}{a^x}$$

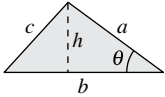
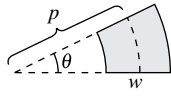
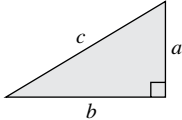
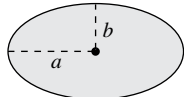
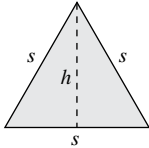
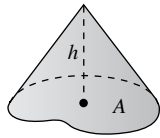
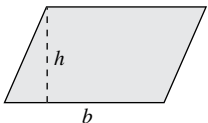
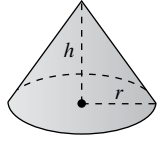
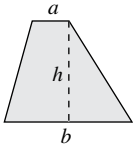
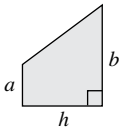
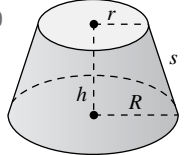
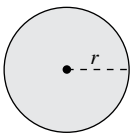
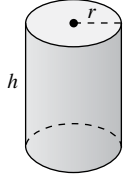
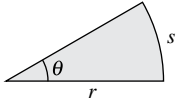
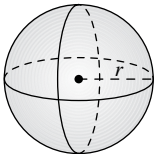
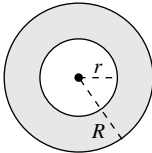
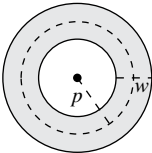
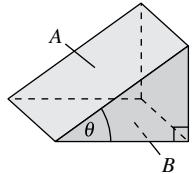
$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$(a^x)^y = a^{xy}$$

$$\sqrt[n]{\sqrt[n]{a}} = \sqrt[n]{a}$$

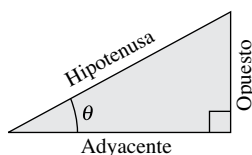
FÓRMULAS TRIGONOMÉTRICAS



Triángulo $h = a \sen \theta$ $\text{Área} = \frac{1}{2}bh$ (Ley de los cosenos) $c^2 = a^2 + b^2 - 2ab \cos \theta$ 	Sector de un anillo circular (p = radio promedio, w = ancho del anillo, θ en radianes) $\text{Área} = \theta pw$ 
Triángulo rectángulo (Teorema de Pitágoras) $c^2 = a^2 + b^2$ 	Elipse $\text{Área} = \pi ab$ $\text{Circunferencia} \approx 2\pi \sqrt{\frac{a^2 + b^2}{2}}$ 
Triángulo equilátero $h = \frac{\sqrt{3}s}{2}$ $\text{Área} = \frac{\sqrt{3}s^2}{4}$ 	Cono (A = área de la base) $\text{Volumen} = \frac{Ah}{3}$ 
Paralelogramo $\text{Área} = bh$ 	Cono circular recto $\text{Volumen} = \frac{\pi r^2 h}{3}$ $\text{Área de la superficie lateral} = \pi r \sqrt{r^2 + h^2}$ 
Trapezoide $\text{Área} = \frac{h}{2}(a + b)$  	Tronco de un cono circular recto $\text{Volumen} = \frac{\pi(r^2 + rR + R^2)h}{3}$ $\text{Área de la superficie lateral} = \pi s(R + r)$ 
Círculo $\text{Área} = \pi r^2$ $\text{Circunferencia} = 2\pi r$ 	Cilindro circular recto $\text{Volumen} = \pi r^2 h$ $\text{Área de la superficie lateral} = 2\pi rh$ 
Sector circular (θ en radianes) $\text{Área} = \frac{\theta r^2}{2}$ $s = r\theta$ 	Esfera $\text{Volumen} = \frac{4}{3}\pi r^3$ $\text{Área de la superficie} = 4\pi r^2$ 
Anillo circular (p = radio promedio, w = ancho del anillo) $\text{Área} = \pi(R^2 - r^2)$ $= 2\pi pw$  	Cuña (A = área de la cara superior, B = área de la base) $A = B \sec \theta$ 

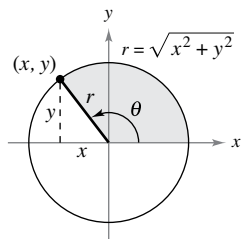
Definiciones de las seis funciones trigonométricas

Definiciones para un triángulo rectángulo, donde $0 < \theta < \pi/2$.

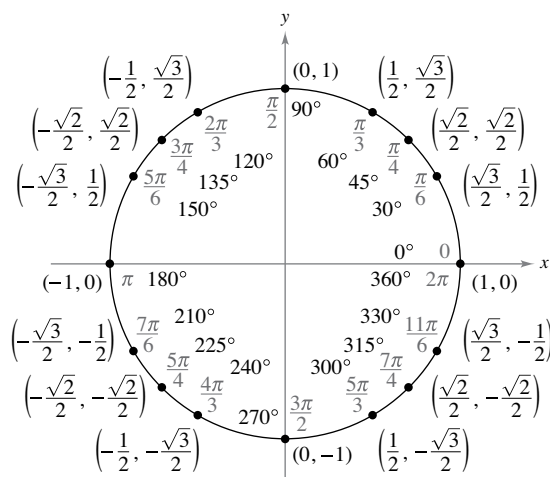


$$\begin{aligned}\sin \theta &= \frac{\text{op}}{\text{hip}} & \csc \theta &= \frac{\text{hip}}{\text{op}} \\ \cos \theta &= \frac{\text{ady}}{\text{hip}} & \sec \theta &= \frac{\text{hip}}{\text{ady}} \\ \tan \theta &= \frac{\text{op}}{\text{ady}} & \cot \theta &= \frac{\text{ady}}{\text{op}}\end{aligned}$$

Definiciones de las funciones circulares, donde θ es cualquier ángulo.



$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$



Identidades recíprocas

$$\begin{aligned}\sin x &= \frac{1}{\csc x} & \sec x &= \frac{1}{\cos x} & \tan x &= \frac{1}{\cot x} \\ \csc x &= \frac{1}{\sin x} & \cos x &= \frac{1}{\sec x} & \cot x &= \frac{1}{\tan x}\end{aligned}$$

Identidades para la tangente y la cotangente

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Identidades pitagóricas

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x & 1 + \cot^2 x &= \csc^2 x\end{aligned}$$

Identidades para cofunciones

$$\begin{aligned}\sin\left(\frac{\pi}{2} - x\right) &= \cos x & \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \csc\left(\frac{\pi}{2} - x\right) &= \sec x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x \\ \sec\left(\frac{\pi}{2} - x\right) &= \csc x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x\end{aligned}$$

Fórmulas de reducción

$$\begin{aligned}\sin(-x) &= -\sin x & \cos(-x) &= \cos x \\ \csc(-x) &= -\csc x & \tan(-x) &= -\tan x \\ \sec(-x) &= \sec x & \cot(-x) &= -\cot x\end{aligned}$$

Fórmulas para la suma y la diferencia

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

Fórmulas para ángulos dobles

$$\begin{aligned}\sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

Fórmulas para la reducción de potencias

$$\begin{aligned}\sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u}\end{aligned}$$

Fórmulas suma a producto

$$\begin{aligned}\sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)\end{aligned}$$

Fórmulas producto a suma

$$\begin{aligned}\sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)]\end{aligned}$$



Reglas básicas de diferenciación

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsen u] = \frac{u'}{\sqrt{1-u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$
23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$
24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u'$
28. $\frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u'$
29. $\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u'$
30. $\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2+1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2-1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1-u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1-u^2}$
35. $\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-u'}{u\sqrt{1-u^2}}$
36. $\frac{d}{dx}[\operatorname{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1+u^2}}$

Fórmulas básicas de integración

1. $\int kf(u) du = k \int f(u) du$
2. $\int [f(u) \pm g(u)] du = \int f(u) du \pm \int g(u) du$
3. $\int du = u + C$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u du = e^u + C$
7. $\int a^u du = \left(\frac{1}{\ln a}\right)a^u + C$
8. $\int \sin u du = -\cos u + C$
9. $\int \cos u du = \sin u + C$
10. $\int \tan u du = -\ln|\cos u| + C$
11. $\int \cot u du = \ln|\sin u| + C$
12. $\int \sec u du = \ln|\sec u + \tan u| + C$
13. $\int \csc u du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u du = \tan u + C$
15. $\int \csc^2 u du = -\cot u + C$
16. $\int \sec u \tan u du = \sec u + C$
17. $\int \csc u \cot u du = -\csc u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsen \frac{u}{a} + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$



TABLAS DE INTEGRACIÓN

Formas que implican u^n

$$1. \int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$2. \int \frac{1}{u} du = \ln|u| + C$$

Formas que implican $a + bu$

$$3. \int \frac{u}{a + bu} du = \frac{1}{b^2}(bu - a \ln|a + bu|) + C$$

$$4. \int \frac{u}{(a + bu)^2} du = \frac{1}{b^2} \left(\frac{a}{a + bu} + \ln|a + bu| \right) + C$$

$$5. \int \frac{u}{(a + bu)^n} du = \frac{1}{b^2} \left[\frac{-1}{(n-2)(a + bu)^{n-2}} + \frac{a}{(n-1)(a + bu)^{n-1}} \right] + C, \quad n \neq 1, 2$$

$$6. \int \frac{u^2}{a + bu} du = \frac{1}{b^3} \left[-\frac{bu}{2}(2a - bu) + a^2 \ln|a + bu| \right] + C$$

$$7. \int \frac{u^2}{(a + bu)^2} du = \frac{1}{b^3} \left(bu - \frac{a^2}{a + bu} - 2a \ln|a + bu| \right) + C$$

$$8. \int \frac{u^2}{(a + bu)^3} du = \frac{1}{b^3} \left[\frac{2a}{a + bu} - \frac{a^2}{2(a + bu)^2} + \ln|a + bu| \right] + C$$

$$9. \int \frac{u^2}{(a + bu)^n} du = \frac{1}{b^3} \left[\frac{-1}{(n-3)(a + bu)^{n-3}} + \frac{2a}{(n-2)(a + bu)^{n-2}} - \frac{a^2}{(n-1)(a + bu)^{n-1}} \right] + C, \quad n \neq 1, 2, 3$$

$$10. \int \frac{1}{u(a + bu)} du = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$$

$$11. \int \frac{1}{u(a + bu)^2} du = \frac{1}{a} \left(\frac{1}{a + bu} + \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| \right) + C$$

$$12. \int \frac{1}{u^2(a + bu)} du = -\frac{1}{a} \left(\frac{1}{u} + \frac{b}{a} \ln \left| \frac{u}{a + bu} \right| \right) + C$$

$$13. \int \frac{1}{u^2(a + bu)^2} du = -\frac{1}{a^2} \left[\frac{a + 2bu}{u(a + bu)} + \frac{2b}{a} \ln \left| \frac{u}{a + bu} \right| \right] + C$$

Formas que implican $a + bu + cu^2, b^2 \neq 4ac$

$$14. \int \frac{1}{a + bu + cu^2} du = \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2cu + b}{\sqrt{4ac - b^2}} + C, & b^2 < 4ac \\ \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2cu + b - \sqrt{b^2 - 4ac}}{2cu + b + \sqrt{b^2 - 4ac}} \right| + C, & b^2 > 4ac \end{cases}$$

$$15. \int \frac{u}{a + bu + cu^2} du = \frac{1}{2c} \left(\ln|a + bu + cu^2| - b \int \frac{1}{a + bu + cu^2} du \right)$$

Formas que implican $\sqrt{a + bu}$

$$16. \int u^n \sqrt{a + bu} du = \frac{2}{b(2n+3)} \left[u^n(a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} du \right]$$

$$17. \int \frac{1}{u \sqrt{a + bu}} du = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, & a > 0 \\ \frac{2}{\sqrt{-a}} \arctan \sqrt{\frac{a + bu}{-a}} + C, & a < 0 \end{cases}$$

$$18. \int \frac{1}{u^n \sqrt{a + bu}} du = \frac{-1}{a(n-1)} \left[\frac{\sqrt{a + bu}}{u^{n-1}} + \frac{(2n-3)b}{2} \int \frac{1}{u^{n-1} \sqrt{a + bu}} du \right], \quad n \neq 1$$

$$19. \int \frac{\sqrt{a+bu}}{u} du = 2\sqrt{a+bu} + a \int \frac{1}{u\sqrt{a+bu}} du$$

$$20. \int \frac{\sqrt{a+bu}}{u^n} du = \frac{-1}{a(n-1)} \left[\frac{(a+bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2} \int \frac{\sqrt{a+bu}}{u^{n-1}} du \right], n \neq 1$$

$$21. \int \frac{u}{\sqrt{a+bu}} du = \frac{-2(2a-bu)}{3b^2} \sqrt{a+bu} + C$$

$$22. \int \frac{u^n}{\sqrt{a+bu}} du = \frac{2}{(2n+1)b} \left(u^n \sqrt{a+bu} - na \int \frac{u^{n-1}}{\sqrt{a+bu}} du \right)$$

Formas que implican $a^2 \pm u^2$, $a > 0$

$$23. \int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$24. \int \frac{1}{u^2 - a^2} du = - \int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$$

$$25. \int \frac{1}{(a^2 \pm u^2)^n} du = \frac{1}{2a^2(n-1)} \left[\frac{u}{(a^2 \pm u^2)^{n-1}} + (2n-3) \int \frac{1}{(a^2 \pm u^2)^{n-1}} du \right], n \neq 1$$

Formas que implican $\sqrt{u^2 \pm a^2}$, $a > 0$

$$26. \int \sqrt{u^2 \pm a^2} du = \frac{1}{2} \left(u \sqrt{u^2 \pm a^2} \pm a^2 \ln |u + \sqrt{u^2 \pm a^2}| \right) + C$$

$$27. \int u^2 \sqrt{u^2 \pm a^2} du = \frac{1}{8} \left[u(2u^2 \pm a^2) \sqrt{u^2 \pm a^2} - a^4 \ln |u + \sqrt{u^2 \pm a^2}| \right] + C$$

$$28. \int \frac{\sqrt{u^2 + a^2}}{u} du = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$29. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \operatorname{arcsec} \frac{|u|}{a} + C$$

$$30. \int \frac{\sqrt{u^2 \pm a^2}}{u^2} du = \frac{-\sqrt{u^2 \pm a^2}}{u} + \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$31. \int \frac{1}{\sqrt{u^2 \pm a^2}} du = \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$32. \int \frac{1}{u\sqrt{u^2 + a^2}} du = \frac{-1}{a} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$33. \int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$

$$34. \int \frac{u^2}{\sqrt{u^2 \pm a^2}} du = \frac{1}{2} \left(u \sqrt{u^2 \pm a^2} \mp a^2 \ln |u + \sqrt{u^2 \pm a^2}| \right) + C$$

$$35. \int \frac{1}{u^2 \sqrt{u^2 \pm a^2}} du = \mp \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$$

$$36. \int \frac{1}{(u^2 \pm a^2)^{3/2}} du = \frac{\pm u}{a^2 \sqrt{u^2 \pm a^2}} + C$$

Formas que implican $\sqrt{a^2 - u^2}$, $a > 0$

$$37. \int \sqrt{a^2 - u^2} du = \frac{1}{2} \left(u \sqrt{a^2 - u^2} + a^2 \arcsen \frac{u}{a} \right) + C$$

$$38. \int u^2 \sqrt{a^2 - u^2} du = \frac{1}{8} \left[u(2u^2 - a^2) \sqrt{a^2 - u^2} + a^4 \arcsen \frac{u}{a} \right] + C$$



$$\begin{aligned} 39. \int \frac{\sqrt{a^2 - u^2}}{u} du &= \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C & 40. \int \frac{\sqrt{a^2 - u^2}}{u^2} du &= \frac{-\sqrt{a^2 - u^2}}{u} - \arcsen \frac{u}{a} + C \\ 41. \int \frac{1}{\sqrt{a^2 - u^2}} du &= \arcsen \frac{u}{a} + C & 42. \int \frac{1}{u\sqrt{a^2 - u^2}} du &= \frac{-1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C \\ 43. \int \frac{u^2}{\sqrt{a^2 - u^2}} du &= \frac{1}{2} \left(-u\sqrt{a^2 - u^2} + a^2 \arcsen \frac{u}{a} \right) + C & 44. \int \frac{1}{u^2\sqrt{a^2 - u^2}} du &= \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C \\ 45. \int \frac{1}{(a^2 - u^2)^{3/2}} du &= \frac{u}{a^2 \sqrt{a^2 - u^2}} + C \end{aligned}$$

Formas que implican $\sin u$ o $\cos u$

$$\begin{aligned} 46. \int \sin u du &= -\cos u + C & 47. \int \cos u du &= \sin u + C \\ 48. \int \sin^2 u du &= \frac{1}{2}(u - \sin u \cos u) + C & 49. \int \cos^2 u du &= \frac{1}{2}(u + \sin u \cos u) + C \\ 50. \int \sin^n u du &= -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n} \int \sin^{n-2} u du & 51. \int \cos^n u du &= \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n} \int \cos^{n-2} u du \\ 52. \int u \sin u du &= \sin u - u \cos u + C & 53. \int u \cos u du &= \cos u + u \sin u + C \\ 54. \int u^n \sin u du &= -u^n \cos u + n \int u^{n-1} \cos u du & 55. \int u^n \cos u du &= u^n \sin u - n \int u^{n-1} \sin u du \\ 56. \int \frac{1}{1 \pm \sin u} du &= \tan u \mp \sec u + C & 57. \int \frac{1}{1 \pm \cos u} du &= -\cot u \pm \csc u + C \\ 58. \int \frac{1}{\sin u \cos u} du &= \ln |\tan u| + C \end{aligned}$$

Formas que implican $\tan u$, $\cot u$, $\sec u$ o $\csc u$

$$\begin{aligned} 59. \int \tan u du &= -\ln |\cos u| + C & 60. \int \cot u du &= \ln |\sin u| + C \\ 61. \int \sec u du &= \ln |\sec u + \tan u| + C \\ 62. \int \csc u du &= \ln |\csc u - \cot u| + C \quad \text{o} \quad \int \csc u du = -\ln |\csc u + \cot u| + C \\ 63. \int \tan^2 u du &= -u + \tan u + C & 64. \int \cot^2 u du &= -u - \cot u + C \\ 65. \int \sec^2 u du &= \tan u + C & 66. \int \csc^2 u du &= -\cot u + C \\ 67. \int \tan^n u du &= \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u du, \quad n \neq 1 & 68. \int \cot^n u du &= -\frac{\cot^{n-1} u}{n-1} - \int (\cot^{n-2} u) du, \quad n \neq 1 \\ 69. \int \sec^n u du &= \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} u du, \quad n \neq 1 \\ 70. \int \csc^n u du &= -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} u du, \quad n \neq 1 \end{aligned}$$



$$71. \int \frac{1}{1 \pm \tan u} du = \frac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$$

$$72. \int \frac{1}{1 \pm \cot u} du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$$

$$73. \int \frac{1}{1 \pm \sec u} du = u + \cot u \mp \csc u + C$$

$$74. \int \frac{1}{1 \pm \csc u} du = u - \tan u \pm \sec u + C$$

Formas que implican funciones trigonométricas

$$75. \int \arcsen u du = u \arcsen u + \sqrt{1-u^2} + C$$

$$76. \int \arccos u du = u \arccos u - \sqrt{1-u^2} + C$$

$$77. \int \arctan u du = u \arctan u - \ln\sqrt{1+u^2} + C$$

$$78. \int \operatorname{arccot} u du = u \operatorname{arccot} u + \ln\sqrt{1+u^2} + C$$

$$79. \int \operatorname{arcsec} u du = u \operatorname{arcsec} u - \ln|u + \sqrt{u^2-1}| + C$$

$$80. \int \operatorname{arccsc} u du = u \operatorname{arccsc} u + \ln|u + \sqrt{u^2-1}| + C$$

Formas que implican e^u

$$81. \int e^u du = e^u + C$$

$$82. \int ue^u du = (u-1)e^u + C$$

$$83. \int u^n e^u du = u^n e^u - n \int u^{n-1} e^u du$$

$$84. \int \frac{1}{1+e^u} du = u - \ln(1+e^u) + C$$

$$85. \int e^{au} \sin bu du = \frac{e^{au}}{a^2+b^2} (a \sin bu - b \cos bu) + C$$

$$86. \int e^{au} \cos bu du = \frac{e^{au}}{a^2+b^2} (a \cos bu + b \sin bu) + C$$

Formas que implican $\ln u$

$$87. \int \ln u du = u(-1 + \ln u) + C$$

$$88. \int u \ln u du = \frac{u^2}{4}(-1 + 2 \ln u) + C$$

$$89. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2}[-1 + (n+1) \ln u] + C, n \neq -1$$

$$90. \int (\ln u)^2 du = u[2 - 2 \ln u + (\ln u)^2] + C$$

$$91. \int (\ln u)^n du = u(\ln u)^n - n \int (\ln u)^{n-1} du$$

Formas que implican funciones hiperbólicas

$$92. \int \cosh u du = \sinh u + C$$

$$93. \int \sinh u du = \cosh u + C$$

$$94. \int \operatorname{sech}^2 u du = \tanh u + C$$

$$95. \int \operatorname{csch}^2 u du = -\coth u + C$$

$$96. \int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$$

$$97. \int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$$

Formas que implican funciones hiperbólicas inversas (en forma logarítmica)

$$98. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$99. \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$

$$100. \int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} \ln \frac{a + \sqrt{a^2 \pm u^2}}{|u|} + C$$